1. A pile of sand in the shape of a cone whose radius is twice its height is growing at a rate of 5 m^3/sec. How fast is its height increasing when the diameter is 40 meters?

Know:

$r = 20\,m$

$h = 10\,m$

$\frac{dV}{dt} = 5\, m^3/s$

$r = 2h$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi (2h)^2 h$$

$$V = \frac{1}{3}\pi 4h^2 h$$

$$V = \frac{4}{3}\pi h^3$$

$$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$$

$$5 = 4\pi (10)^2 \frac{dh}{dt}$$

$$5 = 400\pi \frac{dh}{dt}$$

$$\frac{5}{400\pi} = \frac{dh}{dt}$$

$$\frac{1}{80\pi}\, m/s = \frac{dh}{dt}$$

Need:

$\frac{dh}{dt}$

2. Evaluate: $\int(-2x^{-3}+20x^{-5})dx$

$$= \frac{-2x^{-2}}{-2} + \frac{20x^{-4}}{-4} + c$$

$$= x^{-2} - 5x^{-4} + c$$

$$= \boxed{\frac{1}{x^2} - \frac{5}{x^4} + c}$$

3. Evaluate: $\int\left(\frac{-14x^{5/2}}{2}\right)dx = \int -7x^{5/2}\,dx$

$$= \frac{-7x^{7/2}}{7/2} + c$$

$$= \frac{2}{\cancel{7}} \cdot -\cancel{7}x^{7/2} + c = -2x^{7/2} + c = \boxed{-2\sqrt{x^7} + c}$$

4. Evaluate: $\int \left(\frac{-5\sqrt[3]{x^2}}{3} \right) dx = \int -\frac{5}{3} x^{2/3}\, dx$

$= \frac{-\frac{5}{3} x^{5/3}}{5/3} + c = \cancel{\frac{3}{5}} \cdot -\cancel{\frac{5}{3}} x^{5/3} + c$

$= -x^{5/3} + c = \boxed{-\sqrt[3]{x^5} + c}$

5. Evaluate: $\int_{-1}^{3} (-x^3 + 3x^2 + 1)\, dx$

$= \frac{-x^4}{4} + \frac{3x^3}{3} + x + c = \left. \frac{-x^4}{4} + x^3 + x + c \right]_{-1}^{3}$

$= \left(\frac{-(3)^4}{4} + 3^3 + 3 + \cancel{c} \right) - \left(\frac{-(-1)^4}{4} + (-1)^3 + (-1) + \cancel{c} \right)$

$= \left(-\frac{81}{4} + 30 \right) - \left(-\frac{1}{4} - 1 - 1 \right)$

$= \frac{39}{4} - \left(-\frac{9}{4} \right) = \boxed{12}$

6. Evaluate: $\int_{-3}^{0}\left(4\sqrt[3]{x}\right)dx$ $= \int_{-3}^{0} 4x^{1/3}\,dx$

$= \dfrac{4x^{4/3}}{\frac{4}{3}} + c = \dfrac{3}{\cancel{4}}\cdot \cancel{4}x^{4/3} + c = 3x^{4/3} + c\Big]_{-3}^{0}$

$\left(3(0)^{4/3} + \cancel{c}\right) - \left(3(-3)^{4/3} + \cancel{c}\right) = 0 - \left(3\sqrt[3]{81}\right) = -3\sqrt[3]{81}$

$= \boxed{-9\sqrt[3]{3}}$

7. Given $\dfrac{dy}{dx} = \dfrac{6x^2 - 2x^3}{x}$ and $y(1) = 4$ find y.

$\dfrac{dy}{dx} = 6x - 2x^2$

$y = \dfrac{6x^2}{2} - \dfrac{2x^3}{3} + c$

$y = 3x^2 - \dfrac{2}{3}x^3 + c$

$4 = 3(1)^2 - \dfrac{2}{3}(1)^3 + c$

$4 = 3 - \dfrac{2}{3} + c$

$4 = \dfrac{7}{3} + c$

$-\dfrac{7}{3} \quad -\dfrac{7}{3}$

$\dfrac{5}{3} = c$

$\boxed{y = 3x^2 - \dfrac{2}{3}x^3 + \dfrac{5}{3}}$

8. If $f'(x) = 3x^2 - 8x + 1$ and $f(1) = 4$ find $f(x)$.

$$f(x) = \frac{3x^3}{3} - \frac{8x^2}{2} + x + c$$

$$f(x) = x^3 - 4x^2 + x + c$$

$$4 = 1^3 - 4(1)^2 + 1 + c$$

$$4 = 1 - 4 + 1 + c$$

$$4 = -2 + c$$

$$6 = c$$

$$\boxed{f(x) = x^3 - 4x^2 + x + 6}$$

9. If $f''(x) = 6x^2 - 12x + 2$, $f'(1) = -3$ and $f(-2) = 1$ find $f(x)$.

$$f'(x) = \frac{6x^3}{3} - \frac{12x^2}{2} + 2x + c$$

$$f'(x) = 2x^3 - 6x^2 + 2x + c$$

$$-3 = 2(1)^3 - 6(1)^2 + 2(1) + c$$

$$-3 = 2 - 6 + 2 + c$$

$$-3 = -2 + c$$

$$-1 = c$$

$$f'(x) = 2x^3 - 6x^2 + 2x - 1$$

⊛ $$f(x) = \frac{2x^4}{4} - \frac{6x^3}{3} + \frac{2x^2}{2} - x + d$$

$$= \frac{1}{2}x^4 - 2x^3 + x^2 - x + d$$

$$1 = \frac{1}{2}(-2)^4 - 2(-2)^3 + (-2)^2 - (-2) + d$$

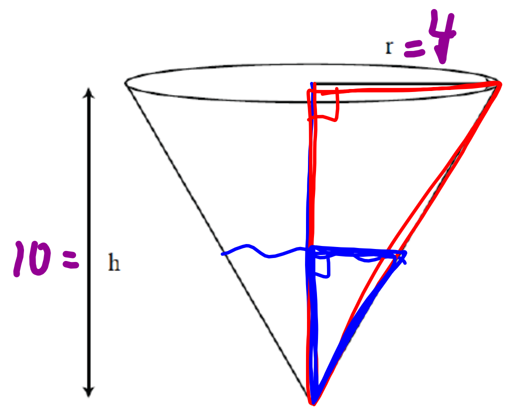
$$1 = 8 + 16 + 4 + 2 + d$$

$$1 = 30 + d$$

$$-29 = d$$

$$\boxed{f(x) = \frac{1}{2}x^4 - 2x^3 + x^2 - x - 29}$$

10. Consider a conical tank whose radius at the top is 4 feet and whose depth is 10 feet. The tank is being filled with water at a rate of $2\,ft^3/\min$. How fast is the water level rising when the depth of the water is 5 feet?



Know:

$r(\text{cone}) = 4\text{ft}$

$h(\text{cone}) = 10\text{ft}$

$\frac{dV}{dt} = 2\,\text{ft}^3/\text{min}$

$h(\text{water}) = 5\text{ft}$

Need: $\frac{dh}{dt}$

$\frac{r}{h} = \frac{4}{10}$ ⊛ solve for "r"

$r = \frac{4}{10}h$

$$V = \frac{1}{3}\pi r^2 h \longrightarrow V = \frac{1}{3}\pi\left(\frac{4}{10}h\right)^2 h$$

$$V = \frac{1}{3}\pi \frac{16}{100}h^2\,h$$

$$V = \frac{16}{300}\pi h^3$$

$\cancel{3} \cdot \frac{16}{\cancel{300}}$ 100

$$\frac{dV}{dt} = \frac{16}{100}\pi h^2 \frac{dh}{dt}$$

$$2 = \frac{16}{100}\pi (5)^2 \frac{dh}{dt}$$

$$2 = \frac{16}{\cancel{100}_{4}}\pi(\cancel{25})\frac{dh}{dt}$$

$$2 = 4\pi \frac{dh}{dt}$$

$$\frac{2}{4\pi} = \frac{dh}{dt}$$

$$\boxed{\frac{1}{2\pi}\,\text{ft}/\text{min} = \frac{dh}{dt}}$$